

# Concepts and Applications of Effective Field Theories: Flavor Physics and Beyond

---

**Matthias Neubert**

Mainz Institute for Theoretical Physics (MITP)  
Johannes Gutenberg University, Mainz, Germany

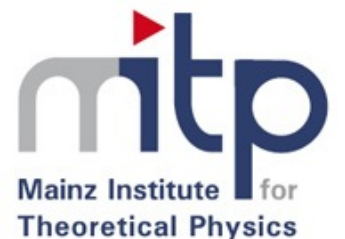


JOHANNES GUTENBERG  
UNIVERSITÄT MAINZ

*BCVSPIN-MSPF-MITCHELL Joint School 2014*

Las Hadas Golf Resort & Marina

Manzanillo, Colima, Mexico, 8-13 December 2014



# Effective Field Theory

---

Effective field theories are a very powerful tool in quantum field theories:

- systematic formalism for the analysis of **multi-scale problems**
- simplifies **practical calculations**, often makes them feasible (“Taylor expansion of Feynman graphs”)
- particularly important in QCD, where short-distance effects are calculable perturbatively, but long-distance effects are not
- provides new perspective on **renormalization**
- basis of **factorization** (i.e. scale separation) and **resummation** of large logarithmic terms

# Effective Field Theory

---

## Useful reviews:

- E. Witten, Nucl. Phys. B 122 (1977) 109
- S. Weinberg, Phys. Lett. B 91 (1980) 51
- L. Hall, Nucl. Phys. B 178 (1981) 75
- J. Polchinsky, hep-th/9210046
- A. Buras, hep-ph/9806471
- M. Neubert, hep-ph/0512222

# Effective Field Theory

---

## **Lecture I:**

- General concepts of EFTs
- Scale separation, integrating out high-energy modes, low-energy effective Lagrangian
- Modern view of QFTs and general principles

## **Lecture II:**

- Applications
- The Standard Model as an effective field theory
- Interesting insights





# **Lecture I: Concepts of Effective Field Theory**



# Derivation of the effective Lagrangian

---

Consider a QFT with a characteristic (fundamental) high-energy scale  $M$

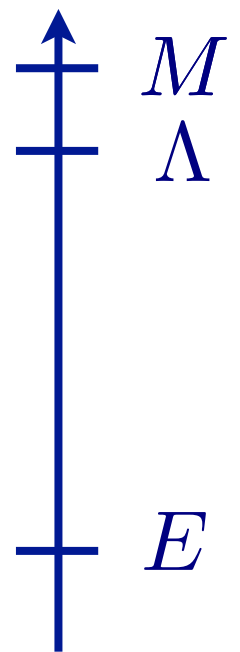
We are interested in performing experiments at energies  $E \ll M$

Step 1: Choose a cutoff  $\Lambda < M$  and divide all quantum fields into high- and low-frequency components ( $\omega > \Lambda$  and  $\omega < \Lambda$ ):

$$\phi = \phi_L + \phi_H$$

Recall:

$$\phi(x) = \int \frac{d^3 k}{(2\pi)^3 2E_k} \left( a_k e^{-ik \cdot x} + a_k^\dagger e^{ik \cdot x} \right)$$



# Derivation of the effective Lagrangian

---

Consider a QFT with a characteristic (fundamental) high-energy scale  $M$

We are interested in performing experiments at energies  $E \ll M$

Step 1: Choose a cutoff  $\Lambda < M$  and divide all quantum fields into high- and low-frequency components ( $\omega > \Lambda$  and  $\omega < \Lambda$ ):

$$\phi = \phi_L + \phi_H$$

Physics (any Green function) at low energies  $E \ll \Lambda$  is entirely described in terms of the fields  $\phi_L$ ; Green functions of these fields can be derived from the generating functional:

$$Z[J_L] = \int \mathcal{D}\phi_L \mathcal{D}\phi_H e^{iS(\phi_L, \phi_H) + i \int d^D x J_L(x) \phi_L(x)}$$

$$\langle 0 | T \{ \phi_L(x_1) \dots \phi_L(x_n) \} | 0 \rangle = \frac{1}{Z[0]} \left( -i \frac{\delta}{\delta J_L(x_1)} \right) \dots \left( -i \frac{\delta}{\delta J_L(x_n)} \right) Z[J_L] \Big|_{J_L=0}$$

# Derivation of the effective Lagrangian

---

Step 2: Since the high-frequency fields  $\phi_H$  do not appear in the generating functional, we can “**integrate them out**” in the path integral:

$$Z[J_L] \equiv \int \mathcal{D}\phi_L e^{iS_\Lambda(\phi_L) + i \int d^D x J_L(x) \phi_L(x)}$$

where

$$e^{iS_\Lambda(\phi_L)} = \int \mathcal{D}\phi_H e^{iS(\phi_L, \phi_H)}$$

and  $S_\Lambda(\phi_L)$  is called the **Wilsonian effective action**

Dependence on the cutoff  $\Lambda$  enters via the condition on the frequencies of the fields

# Derivation of the effective Lagrangian

---

Step 3: Effective action is **non-local** on the scale  $\Delta t \sim 1/\omega$ , corresponding to the propagation of high-energy modes that have been removed from the Lagrangian

Since the remaining fields have energies  $\omega < \Lambda$ , the non-local effective action can be expanded in an **infinite series of local operators**:

$$S_{\Lambda}(\phi_L) = \int d^D x \mathcal{L}_{\Lambda}^{\text{eff}}(x)$$

where:

$$\mathcal{L}_{\Lambda}^{\text{eff}}(x) = \sum_i g_i Q_i(\phi_L(x))$$

coupling constants  
(Wilson coefficients)

local operators built out of  
fields  $\phi_L$  and their derivatives

# Dimensional analysis

---

Does a Lagrangian consisting of an infinite number of interactions and hence an infinite number of (renormalized) coupling constants give us any predictive power?

- Not if one adopt an old-fashioned view about renormalization and renormalizable QFTs
- But not all is lost...

Can use **naive dimensional analysis** to estimate the size of individual terms in the infinite sum to a given matrix element

# Dimensional analysis

---

As is common practice in particle physics, we adopt units where  $\hbar = c = 1$ , such that  $[m] = [E] = [p] = [x^{-1}] = [t^{-1}]$  are all measured in the same units (mass units)

Denote by  $[g_i] = -\gamma_i$  the mass dimension of the coupling constants in the effective Lagrangian

Since by assumption the theory has only a single fundamental scale  $M$ , it follows that:

$$g_i = C_i M^{-\gamma_i}$$

where by **naturalness** we expect that  $C_i = O(1)$

# Dimensional analysis

---

At low energy, it follows that the contribution of a given term  $g_i Q_i$  to an observable (which for simplicity we assume to be dimensionless) scales like:

$$C_i \left( \frac{E}{M} \right)^{\gamma_i} = \begin{cases} O(1) ; & \text{if } \gamma_i = 0 \\ \ll 1 ; & \text{if } \gamma_i > 0 \\ \gg 1 ; & \text{if } \gamma_i < 0 \end{cases}$$

Therefore, only operators with  $\gamma_i \leq 0$  are important for  $E \ll M$

This is what makes the effective Lagrangian useful!

Depending on the precision goal, one can truncate the infinite sum over terms by only retaining operators whose  $\gamma_i$  value is smaller than a certain value



# Dimensional analysis

---

Since the Lagrangian has mass dimension  $D$  = dimensionality of spacetime (the action is dimensionless), it follows that

$$\delta_i = [Q_i] = D + \gamma_i$$

Hence we can summarize:

Dimension	Importance for $E \rightarrow 0$	Terminology
$\delta_i < D, \gamma_i < 0$	grows	relevant operators (super-renormalizable)
$\delta_i = D, \gamma_i = 0$	constant	marginal operators (renormalizable)
$\delta_i > D, \gamma_i > 0$	falls	irrelevant operators (non-renormalizable)

Only a **finite number** of relevant and marginal operators exist!

# Dimensional analysis

---

Comments:

Dimension	Importance for $E \rightarrow 0$	Terminology
$\delta_i < D, \gamma_i < 0$	grows	relevant operators (super-renormalizable)
$\delta_i = D, \gamma_i = 0$	constant	marginal operators (renormalizable)
$\delta_i > D, \gamma_i > 0$	falls	irrelevant operators (non-renormalizable)

- “**relevant**” operators are usually unimportant, since they are forbidden by some symmetry (otherwise they give rise to a hierarchy problem)
- “**marginal**” operators are all there is in renormalizable QFTs
- “**irrelevant**” operators are the most interesting ones, since they tell us something about the fundamental scale  $M$

# Example: $\phi^4$ - theory at weak coupling

---

Use the free Lagrangian to derive the mass dimension of all fields and couplings, assuming the theory is weakly coupled:

$$S = \int d^D x \left( \frac{1}{2} \partial_\mu \phi \partial^\mu \phi - \frac{m^2}{2} \phi^2 - \frac{\lambda}{4!} \phi^4 \right)$$

In  $D$  dimensions, it follows that:

$$[\phi] = \frac{D}{2} - 1, \quad [m] = 1, \quad [\lambda] = 4 - D$$

Hence:

- The mass term is a relevant operator
- The interaction term is marginal in  $D=4$  (relevant in  $D<4$ )

# Example: $\phi^4$ - theory at weak coupling

---

Use the free Lagrangian to derive the mass dimension of all fields and couplings, assuming the theory is weakly coupled:

$$S = \int d^D x \left( \frac{1}{2} \partial_\mu \phi \partial^\mu \phi - \frac{m^2}{2} \phi^2 - \frac{\lambda}{4!} \phi^4 \right)$$

In  $D$  dimensions, it follows that:

$$[\phi] = \frac{D}{2} - 1, \quad [m] = 1, \quad [\lambda] = 4 - D$$

Hence:

- An operator containing  $n_1$  fields  $\phi$  and  $n_2$  derivatives has dimension:

$$\delta_i = n_1 \left( \frac{D}{2} - 1 \right) + n_2, \quad \gamma_i = (n_1 - 2) \left( \frac{D}{2} - 1 \right) + (n_2 - 2)$$

- For  $D > 2$ , adding fields or derivatives increases the dimension!

# Comments

---

Examples of effective field theories:

High-energy theory	Fundamental scale	Low-energy theory
Standard Model	$M_W \sim 80 \text{ GeV}$	Fermi theory
GUT	$M_{\text{GUT}} \sim 10^{16} \text{ GeV}$	Standard Model
String theory	$M_S \sim 10^{18} \text{ GeV}$	QFT
11-dim. $M$ theory	...	String theory
...	...	...
QCD	$m_b \sim 5 \text{ GeV}$	HQET, NRQCD
	$M_{\text{ChSM}} \sim 1 \text{ GeV}$	ChPT

- SM and GUTs are perturbative QFTs
- Fermi theory contains only irrelevant operators (4 fermions)
- String/M theory: fundamental theory is non-local and even spacetime breaks down at short distances

# Comments

---

Examples of effective field theories:

High-energy theory	Fundamental scale	Low-energy theory
Standard Model	$M_W \sim 80 \text{ GeV}$	Fermi theory
GUT	$M_{\text{GUT}} \sim 10^{16} \text{ GeV}$	Standard Model
String theory	$M_S \sim 10^{18} \text{ GeV}$	QFT
11-dim. $M$ theory	...	String theory
...	...	...
QCD	$m_b \sim 5 \text{ GeV}$	HQET, NRQCD
	$M_{\text{ChSM}} \sim 1 \text{ GeV}$	ChPT

- QCD at low energy: example with strong coupling, where the relevant degrees of freedom at low energy (hadrons) are different from the degrees of freedom of QCD
- Low-energy theory is strongly coupled, yet ChPT is useful

# Running couplings / Wilson coefficients

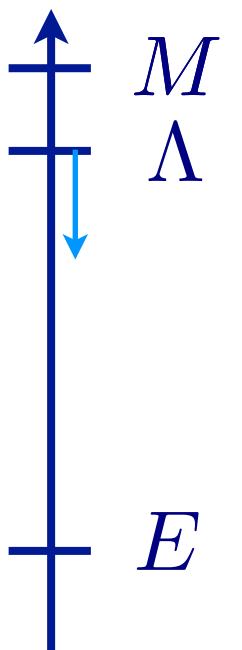
---

Often the fields  $\phi_H$  correspond to heavy particles, whose effects become unimportant at low energies

But the frequency decomposition implies that **high-energy excitations of massless particles** (such as gauge bosons) are also integrated out from the low-energy effective theory

Consider now the situation where we lower the cutoff  $\Lambda$  without crossing the threshold for a heavy particle that could be integrated out:

- the structure of the operators  $Q_i$  in the effective Lagrangian remains the same
- hence, the effect of lowering the cutoff must be entirely absorbed into the values of the coupling constants  $g_i$



Follows that  $g_i = g_i(\Lambda)$  are **running**,  $\Lambda$ -dependent parameters!

# Modern quantum field theory

---

“Theorem of modesty”:

- no QFT ever is complete on all length and energy scales
- all QFTs are low-energy effective theories valid in some energy range, up to some cutoff  $\Lambda$

Giving up renormalizability as a construction criterion for “decent” QFTs:

- at low energy, any effective theory will automatically reduce to a “renormalizable” QFT, meaning that “non-renormalizable” interactions give rise to small contributions  $\sim (E/M)^n$
- this does not make renormalization irrelevant, but it provides a different point of view (Wilsonian picture of the RG)



# Modern quantum field theory

---

We should forget the folklore about “cancellations of infinities”

Adopt the more physical viewpoint that:

- low-energy physics depends on the **short-distance dynamics** of the fundamental theory only through a small number of **relevant and marginal couplings**, and possibly through some irrelevant couplings if our measurements are sufficiently precise
- this finite number of couplings can be renormalized (i.e., infinities can be removed consistently) using a finite number of experimental data
- the criterion of “renormalizability” is **automatically fulfilled** (approximately) by any effective field theory

# Modern quantum field theory

---

We should forget the folklore about “cancellations of infinities”

Adopt the more physical viewpoint that:

- contrary to the old paradigm of strictly forbidding irrelevant interactions, we always expect them to be present and give rise to small effects, which may or may not be observable at a given level of accuracy
- this provides an “**indirect way**” to search for hints of physics beyond the (current) Standard Model:

**low-energy, high-precision measurements**

- e.g.: flavor physics, neutrino physics,  $(g-2)_\mu$ , EDMs, dark-photon searches, ...

# Modern quantum field theory

---

Instead, relevant (“super-renormalizable”) interactions cause problems!

Consider, e.g., the mass term  $m^2 \phi^2$  in scalar field theory

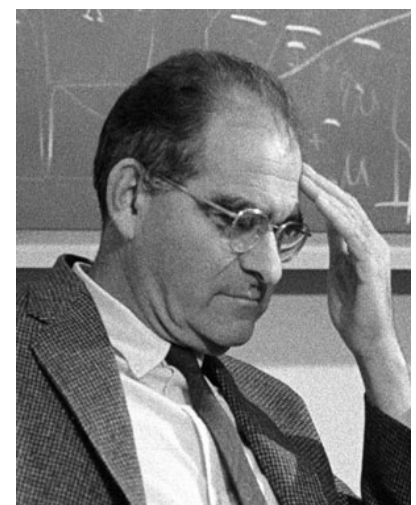
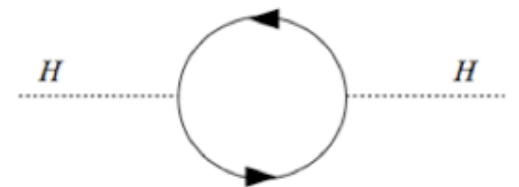
Dimensional analysis suggests that  $m^2 \sim M^2 \sim \Lambda_{\text{UV}}^2$

But then a light scalar particle should not be present in the low-energy effective theory!

**Hierarchy problem!**

The same argument applies for all mass terms in any QFT!

And likewise for the cosmological constant!



Victor Weisskopf

# Modern quantum field theory

---

New paradigm: EFTs must be **natural** in the sense that **all mass terms should be forbidden** by (exact or broken) symmetries!

Indeed:

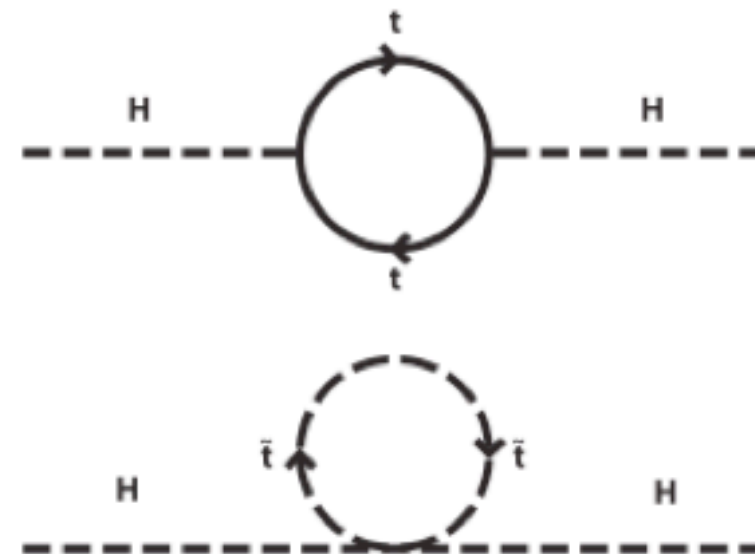
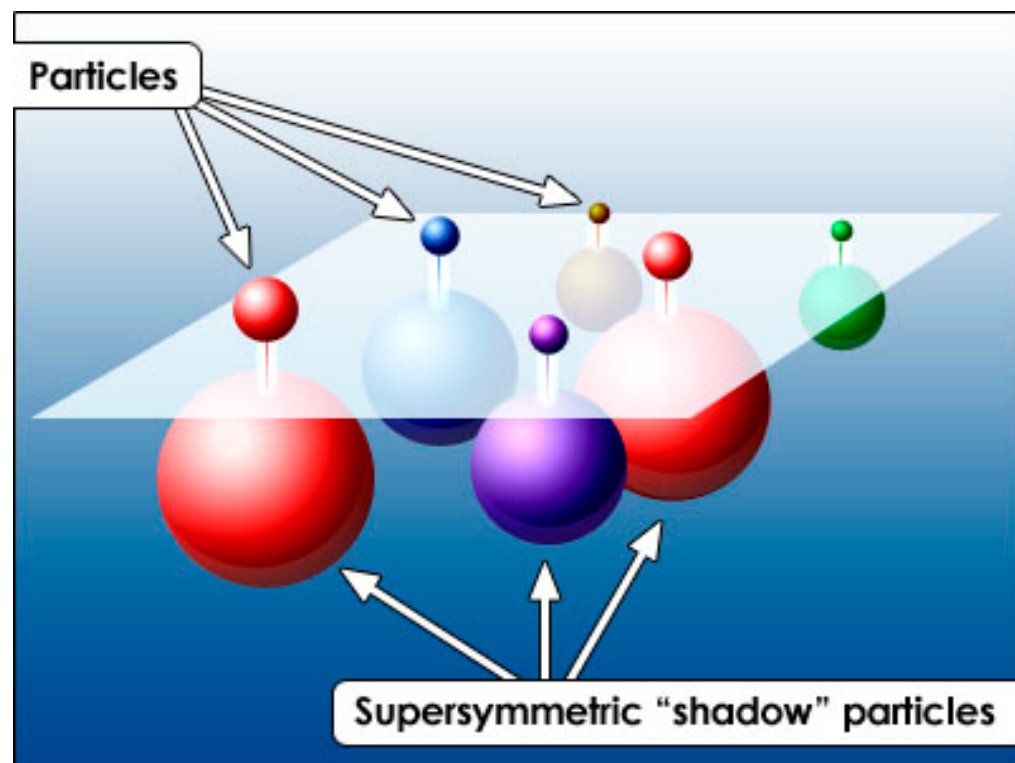
- **gauge invariance:** forbids mass terms for gauge fields (photons and gluons in the Standard Model)
- **chiral symmetry:** forbids mass terms for fermions (all matter fields in the Standard Model)

**Explains why the SM is a (broken) chiral gauge theory!**

- But the **Higgs boson** exists and causes a naturalness problem!
- **Supersymmetry:** would link the masses of scalars and fermions and hence, in combination with chiral symmetry, forbid mass terms for scalar fields (solves hierarchy problem)

# Is nature supersymmetric?

- The **Higgs boson** exists and causes a naturalness problem!
- **Supersymmetry**: would link the masses of scalars and fermions and hence, in combination with chiral symmetry, forbid mass terms for scalar fields (solves hierarchy problem)



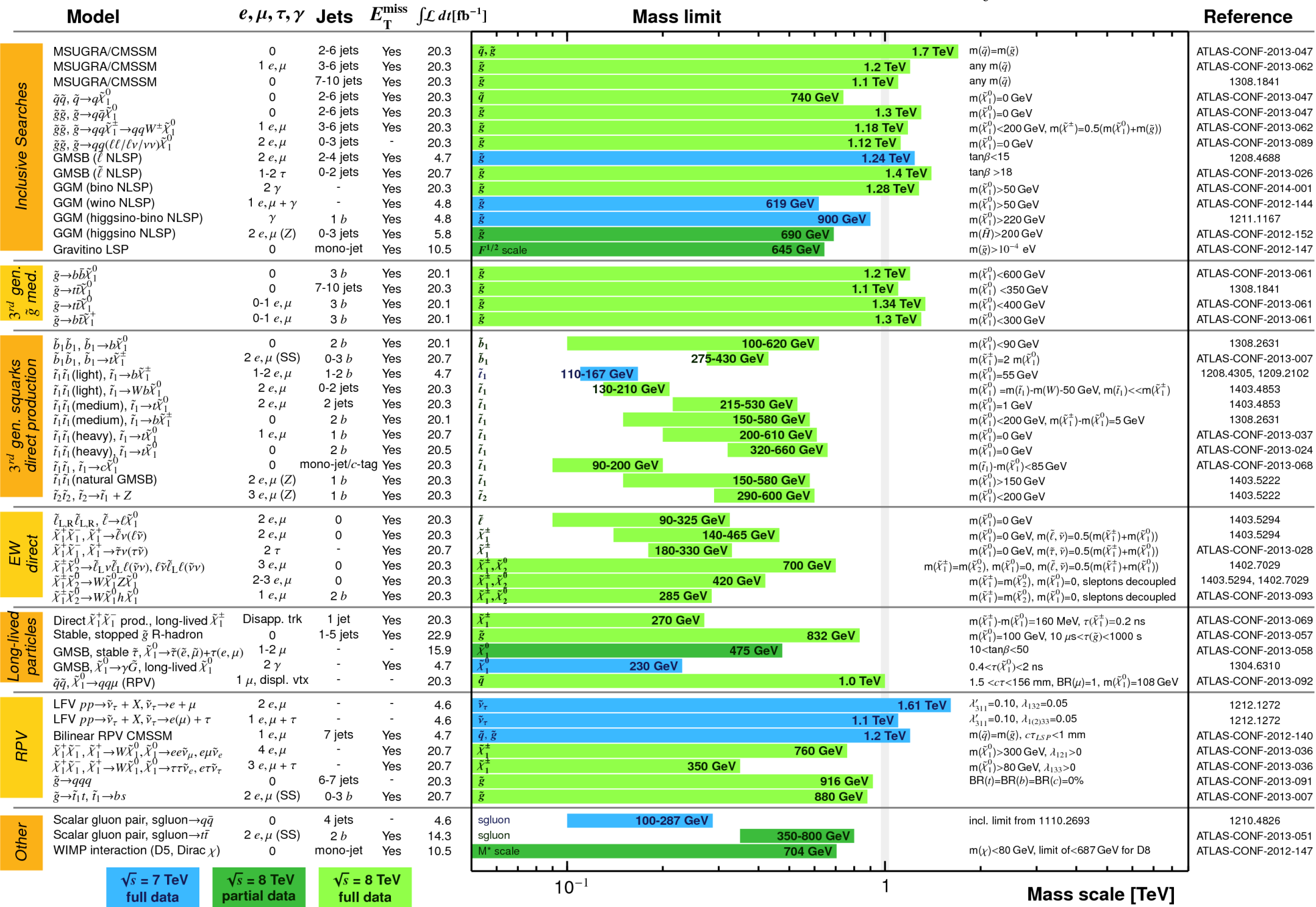
# Is nature supersymmetric?

## ATLAS SUSY Searches\* - 95% CL Lower Limits

Status: Moriond 2014

ATLAS Preliminary

$\int \mathcal{L} dt = (4.6 - 22.9) \text{ fb}^{-1}$   $\sqrt{s} = 7, 8 \text{ TeV}$



\*Only a selection of the available mass limits on new states or phenomena is shown. All limits quoted are observed minus 1 $\sigma$  theoretical signal cross section uncertainty.





## Lecture II: Some applications of EFTs



# Standard Model as an effective field theory

---

Some interesting insights can be gained by considering the Standard Model (SM) as a low-energy effective theory of some more fundamental theory (supersymmetry, extra dimensions, new strongly coupled physics, GUT, ...)

We will denote the **scale of New Physics** by  $M$ ; this could be as large as  $10^{16}$  GeV for some applications, but as small as  $10^3$  GeV (= 1 TeV) for others

The SM Lagrangian should then be extended to an effective Lagrangian, which besides the SM terms contains **additional, irrelevant operators**

These operators must **respect the symmetries of the SM** (gauge invariance, Lorentz symmetry, CPT) but are otherwise unrestricted



# Standard Model as an effective field theory

---

The effective Lagrangian encoding BSM effects up to operator dimension  $d=6$  reads:

$$\mathcal{L}_{SM} = \mathcal{L}_{SM}^{(4)} + \frac{1}{\Lambda} \sum_k C_k^{(5)} Q_k^{(5)} + \frac{1}{\Lambda^2} \sum_k C_k^{(6)} Q_k^{(6)} + \mathcal{O}\left(\frac{1}{\Lambda^3}\right)$$

unique operator (neutrino masses):

$$Q_{\nu\nu} = (\tilde{\varphi}^\dagger l_p)^T C (\tilde{\varphi}^\dagger l_r)$$

Weinberg (1979)

# Standard Model as an effective field theory

The effective Lagrangian encoding BSM effects up to operator dimension d=6 reads:

$$\mathcal{L}_{SM} = \mathcal{L}_{SM}^{(4)} + \frac{1}{\Lambda} \sum_k C_k^{(5)} Q_k^{(5)} + \frac{1}{\Lambda^2} \sum_k C_k^{(6)} Q_k^{(6)} + \mathcal{O}\left(\frac{1}{\Lambda^3}\right)$$

$X^3$		$\varphi^6$ and $\varphi^4 D^2$		$\psi^2 \varphi^3$	
$Q_G$	$f^{ABC} G_\mu^{A\nu} G_\nu^{B\rho} G_\rho^{C\mu}$	$Q_\varphi$	$(\varphi^\dagger \varphi)^3$	$Q_{e\varphi}$	$(\varphi^\dagger \varphi)(\bar{l}_p e_r \varphi)$
$Q_{\tilde{G}}$	$f^{ABC} \tilde{G}_\mu^{A\nu} G_\nu^{B\rho} G_\rho^{C\mu}$	$Q_{\varphi\Box}$	$(\varphi^\dagger \varphi)\Box(\varphi^\dagger \varphi)$	$Q_{u\varphi}$	$(\varphi^\dagger \varphi)(\bar{q}_p u_r \tilde{\varphi})$
$Q_W$	$\varepsilon^{IJK} W_\mu^{I\nu} W_\nu^{J\rho} W_\rho^{K\mu}$	$Q_{\varphi D}$	$(\varphi^\dagger D^\mu \varphi)^* (\varphi^\dagger D_\mu \varphi)$	$Q_{d\varphi}$	$(\varphi^\dagger \varphi)(\bar{q}_p d_r \varphi)$
$Q_{\tilde{W}}$	$\varepsilon^{IJK} \tilde{W}_\mu^{I\nu} W_\nu^{J\rho} W_\rho^{K\mu}$				
$X^2 \varphi^2$		$\psi^2 X \varphi$		$\psi^2 \varphi^2 D$	
$Q_{\varphi G}$	$\varphi^\dagger \varphi G_{\mu\nu}^A G^{A\mu\nu}$	$Q_{eW}$	$(\bar{l}_p \sigma^{\mu\nu} e_r) \tau^I \varphi W_{\mu\nu}^I$	$Q_{\varphi l}^{(1)}$	$(\varphi^\dagger i \overleftrightarrow{D}_\mu \varphi)(\bar{l}_p \gamma^\mu l_r)$
$Q_{\varphi \tilde{G}}$	$\varphi^\dagger \varphi \tilde{G}_{\mu\nu}^A G^{A\mu\nu}$	$Q_{eB}$	$(\bar{l}_p \sigma^{\mu\nu} e_r) \varphi B_{\mu\nu}$	$Q_{\varphi l}^{(3)}$	$(\varphi^\dagger i \overleftrightarrow{D}_\mu^I \varphi)(\bar{l}_p \tau^I \gamma^\mu l_r)$
$Q_{\varphi W}$	$\varphi^\dagger \varphi W_{\mu\nu}^I W^{I\mu\nu}$	$Q_{uG}$	$(\bar{q}_p \sigma^{\mu\nu} T^A u_r) \tilde{\varphi} G_{\mu\nu}^A$	$Q_{\varphi e}$	$(\varphi^\dagger i \overleftrightarrow{D}_\mu \varphi)(\bar{e}_p \gamma^\mu e_r)$
$Q_{\varphi \tilde{W}}$	$\varphi^\dagger \varphi \tilde{W}_{\mu\nu}^I W^{I\mu\nu}$	$Q_{uW}$	$(\bar{q}_p \sigma^{\mu\nu} u_r) \tau^I \tilde{\varphi} W_{\mu\nu}^I$	$Q_{\varphi q}^{(1)}$	$(\varphi^\dagger i \overleftrightarrow{D}_\mu \varphi)(\bar{q}_p \gamma^\mu q_r)$
$Q_{\varphi B}$	$\varphi^\dagger \varphi B_{\mu\nu} B^{\mu\nu}$	$Q_{uB}$	$(\bar{q}_p \sigma^{\mu\nu} u_r) \tilde{\varphi} B_{\mu\nu}$	$Q_{\varphi q}^{(3)}$	$(\varphi^\dagger i \overleftrightarrow{D}_\mu^I \varphi)(\bar{q}_p \tau^I \gamma^\mu q_r)$
$Q_{\varphi \tilde{B}}$	$\varphi^\dagger \varphi \tilde{B}_{\mu\nu} B^{\mu\nu}$	$Q_{dG}$	$(\bar{q}_p \sigma^{\mu\nu} T^A d_r) \varphi G_{\mu\nu}^A$	$Q_{\varphi u}$	$(\varphi^\dagger i \overleftrightarrow{D}_\mu \varphi)(\bar{u}_p \gamma^\mu u_r)$
$Q_{\varphi WB}$	$\varphi^\dagger \tau^I \varphi W_{\mu\nu}^I B^{\mu\nu}$	$Q_{dW}$	$(\bar{q}_p \sigma^{\mu\nu} d_r) \tau^I \varphi W_{\mu\nu}^I$	$Q_{\varphi d}$	$(\varphi^\dagger i \overleftrightarrow{D}_\mu \varphi)(\bar{d}_p \gamma^\mu d_r)$
$Q_{\varphi \tilde{W}B}$	$\varphi^\dagger \tau^I \varphi \tilde{W}_{\mu\nu}^I B^{\mu\nu}$	$Q_{dB}$	$(\bar{q}_p \sigma^{\mu\nu} d_r) \varphi B_{\mu\nu}$	$Q_{\varphi ud}$	$i(\tilde{\varphi}^\dagger D_\mu \varphi)(\bar{u}_p \gamma^\mu d_r)$

59 operator (2499 incl. flavor q. numbers)

Buchmüller, Wyler (1986)

Hagiwara et al. (1987 & 1993)

Grzadkowski, Iskrzynski, Misiak, Rosiek (2010)

Operators other than four-fermion operators

# Standard Model as an effective field theory

The effective Lagrangian encoding BSM effects up to operator dimension d=6 reads:

$$\mathcal{L}_{SM} = \mathcal{L}_{SM}^{(4)} + \frac{1}{\Lambda} \sum_k C_k^{(5)} Q_k^{(5)} + \frac{1}{\Lambda^2} \sum_k C_k^{(6)} Q_k^{(6)} + \mathcal{O}\left(\frac{1}{\Lambda^3}\right)$$

$(\bar{L}L)(\bar{L}L)$		$(\bar{R}R)(\bar{R}R)$		$(\bar{L}L)(\bar{R}R)$	
$Q_{ll}$	$(\bar{l}_p \gamma_\mu l_r)(\bar{l}_s \gamma^\mu l_t)$	$Q_{ee}$	$(\bar{e}_p \gamma_\mu e_r)(\bar{e}_s \gamma^\mu e_t)$	$Q_{le}$	$(\bar{l}_p \gamma_\mu l_r)(\bar{e}_s \gamma^\mu e_t)$
$Q_{qq}^{(1)}$	$(\bar{q}_p \gamma_\mu q_r)(\bar{q}_s \gamma^\mu q_t)$	$Q_{uu}$	$(\bar{u}_p \gamma_\mu u_r)(\bar{u}_s \gamma^\mu u_t)$	$Q_{lu}$	$(\bar{l}_p \gamma_\mu l_r)(\bar{u}_s \gamma^\mu u_t)$
$Q_{qq}^{(3)}$	$(\bar{q}_p \gamma_\mu \tau^I q_r)(\bar{q}_s \gamma^\mu \tau^I q_t)$	$Q_{dd}$	$(\bar{d}_p \gamma_\mu d_r)(\bar{d}_s \gamma^\mu d_t)$	$Q_{ld}$	$(\bar{l}_p \gamma_\mu l_r)(\bar{d}_s \gamma^\mu d_t)$
$Q_{lq}^{(1)}$	$(\bar{l}_p \gamma_\mu l_r)(\bar{q}_s \gamma^\mu q_t)$	$Q_{eu}$	$(\bar{e}_p \gamma_\mu e_r)(\bar{u}_s \gamma^\mu u_t)$	$Q_{qe}$	$(\bar{q}_p \gamma_\mu q_r)(\bar{e}_s \gamma^\mu e_t)$
$Q_{lq}^{(3)}$	$(\bar{l}_p \gamma_\mu \tau^I l_r)(\bar{q}_s \gamma^\mu \tau^I q_t)$	$Q_{ed}$	$(\bar{e}_p \gamma_\mu e_r)(\bar{d}_s \gamma^\mu d_t)$	$Q_{qu}^{(1)}$	$(\bar{q}_p \gamma_\mu q_r)(\bar{u}_s \gamma^\mu u_t)$
		$Q_{ud}^{(1)}$	$(\bar{u}_p \gamma_\mu u_r)(\bar{d}_s \gamma^\mu d_t)$	$Q_{qu}^{(8)}$	$(\bar{q}_p \gamma_\mu T^A q_r)(\bar{u}_s \gamma^\mu T^A u_t)$
		$Q_{ud}^{(8)}$	$(\bar{u}_p \gamma_\mu T^A u_r)(\bar{d}_s \gamma^\mu T^A d_t)$	$Q_{qd}^{(1)}$	$(\bar{q}_p \gamma_\mu q_r)(\bar{d}_s \gamma^\mu d_t)$
				$Q_{qd}^{(8)}$	$(\bar{q}_p \gamma_\mu T^A q_r)(\bar{d}_s \gamma^\mu T^A d_t)$
$(\bar{L}R)(\bar{R}L)$ and $(\bar{L}R)(\bar{L}R)$		B-violating			
$Q_{ledq}$	$(\bar{l}_p^j e_r)(\bar{d}_s^j q_t^j)$	$Q_{duq}$	$\varepsilon^{\alpha\beta\gamma} \varepsilon_{jk} [(d_p^\alpha)^T C u_r^\beta] [(q_s^\gamma)^T C l_t^k]$		
$Q_{quqd}^{(1)}$	$(\bar{q}_p^j u_r) \varepsilon_{jk} (\bar{q}_s^k d_t)$	$Q_{qqqu}$	$\varepsilon^{\alpha\beta\gamma} \varepsilon_{jk} [(q_p^\alpha)^T C q_r^{\beta k}] [(u_s^\gamma)^T C e_t]$		
$Q_{quqd}^{(8)}$	$(\bar{q}_p^j T^A u_r) \varepsilon_{jk} (\bar{q}_s^k T^A d_t)$	$Q_{qqq}^{(1)}$	$\varepsilon^{\alpha\beta\gamma} \varepsilon_{jk} \varepsilon_{mn} [(q_p^\alpha)^T C q_r^{\beta k}] [(q_s^\gamma)^T C l_t^m]$		
$Q_{lequ}^{(1)}$	$(\bar{l}_p^j e_r) \varepsilon_{jk} (\bar{q}_s^k u_t)$	$Q_{qqq}^{(3)}$	$\varepsilon^{\alpha\beta\gamma} (\tau^I \varepsilon)_{jk} (\tau^I \varepsilon)_{mn} [(q_p^\alpha)^T C q_r^{\beta k}] [(q_s^\gamma)^T C l_t^m]$		
$Q_{lequ}^{(3)}$	$(\bar{l}_p^j \sigma_{\mu\nu} e_r) \varepsilon_{jk} (\bar{q}_s^k \sigma^{\mu\nu} u_t)$	$Q_{duu}$	$\varepsilon^{\alpha\beta\gamma} [(d_p^\alpha)^T C u_r^\beta] [(u_s^\gamma)^T C e_t]$		

Four-fermion operators

59 operator (2499 incl. flavor q. numbers)

Buchmüller, Wyler (1986)

Hagiwara et al. (1987 & 1993)

Grzadkowski, Iskrzynski, Misiak, Rosiek (2010)

Flavor observables are crucial in order to explore this enormous parameter space!

# Standard Model as an effective field theory

---

We will discuss a couple of interesting aspects of SM physics from the perspective of this constructions:

- neutrino masses and the see-saw mechanism
- effective weak interactions in the quark sector
- anomalous magnetic moment of the muon
- proton decay
- conservation of baryon and lepton numbers (accidental symmetries)
- Higgs production at the LHC

# Neutrino masses and the see-saw mechanism

---

The discovery of **non-zero neutrino masses** is often described as a departure from the SM

But this is no longer true if we consider the SM as an effective low-energy theory

**Without a right-handed neutrino** (which indeed is not part of the SM), it is impossible to write a neutrino mass term at the level of relevant or marginal operators

However, it **is** possible to write a gauge-invariant **neutrino mass term** at the level of **irrelevant operators** of dimension  $\geq 5$ :

$$\mathcal{L}_{\text{neutrino mass}} = \frac{g}{M} (\tilde{l}_L^T \Phi^*) C (\tilde{\Phi} l_L)$$

# Neutrino masses and the see-saw mechanism

---

However, it **is** possible to write a gauge-invariant **neutrino mass term** at the level of **irrelevant operators** of dimension  $\geq 5$ :

$$\mathcal{L}_{\text{neutrino mass}} = \frac{g}{M} (\tilde{l}_L^T \Phi^*) C (\tilde{\Phi} l_L)$$

After electroweak symmetry breaking, this gives rise to a Majorana mass term of the form:

$$\mathcal{L}_{\text{neutrino mass}} = -\frac{v^2 g}{2M} \tilde{\nu}_L^T C \nu_L$$

The SM as an effective field theory **predicts** that neutrinos should be massive, with  $m_\nu \sim v^2/M$  suppressed by the fundamental scale of some BSM physics

# Neutrino masses and the see-saw mechanism

---

Experiments hints at the fact that the fundamental scale relevant for the generation of neutrino masses is very heavy,

$$M \sim 10^{14} \text{ GeV}$$

which is not far from the scale of grand unification

Extensions of the SM containing **heavy, right-handed neutrinos** (with masses that are naturally of order  $M$ ) provide explicit examples of fundamental theories which yield such a Majorana mass term when the heavy, right-handed neutrinos are integrated out (**see-saw mechanism**)



# Weak interactions at low energies (flavor physics)

---

Fermi's description of the weak interactions at low energy is a prime example of an effective field theory, which has provided first evidence for the **scale of electroweak symmetry breaking**

At the low energies relevant for neutron  $\beta$ -decay, kaon physics, charm physics or B-meson physics (few MeV - few GeV), we can integrate out the heavy  $W$  and  $Z$  bosons as well as the top-quark and Higgs boson from the SM

This gives rise to a low-energy effective theory containing **4-fermion interactions** (Fermi theory) and **dipole interactions** between fermions and the photon and gluon

This effective Lagrangian successfully describes the huge phenomenology of **flavor-changing processes**



# Weak interactions at low energies (flavor physics)

Example: Effective Lagrangian for  $b \rightarrow s$  FCNC transitions  
(see Buras lectures for a derivation)

$$\lambda_p = V_{ps}^* V_{pb} \quad (\text{CKM matrix elements})$$

$$\mathcal{H}_{\text{eff}} = \frac{G_F}{\sqrt{2}} \sum_{p=u,c} \lambda_p \left( C_1 Q_1^p + C_2 Q_2^p + \sum_{i=3,\dots,10} C_i Q_i + C_{7\gamma} Q_{7\gamma} + C_{8g} Q_{8g} \right)$$

$$Q_1^p = (\bar{p}b)_{V-A} (\bar{s}p)_{V-A},$$

$$Q_2^p = (\bar{p}_i b_j)_{V-A} (\bar{s}_j p_i)_{V-A},$$

$$Q_3 = (\bar{s}b)_{V-A} \sum_q (\bar{q}q)_{V-A},$$

$$Q_4 = (\bar{s}_i b_j)_{V-A} \sum_q (\bar{q}_j q_i)_{V-A},$$

$$Q_5 = (\bar{s}b)_{V-A} \sum_q (\bar{q}q)_{V+A},$$

$$Q_6 = (\bar{s}_i b_j)_{V-A} \sum_q (\bar{q}_j q_i)_{V+A},$$

$$Q_7 = (\bar{s}b)_{V-A} \sum_q \frac{3}{2} e_q (\bar{q}q)_{V+A},$$

$$Q_8 = (\bar{s}_i b_j)_{V-A} \sum_q \frac{3}{2} e_q (\bar{q}_j q_i)_{V+A},$$

$$Q_9 = (\bar{s}b)_{V-A} \sum_q \frac{3}{2} e_q (\bar{q}q)_{V-A},$$

$$Q_{10} = (\bar{s}_i b_j)_{V-A} \sum_q \frac{3}{2} e_q (\bar{q}_j q_i)_{V-A},$$

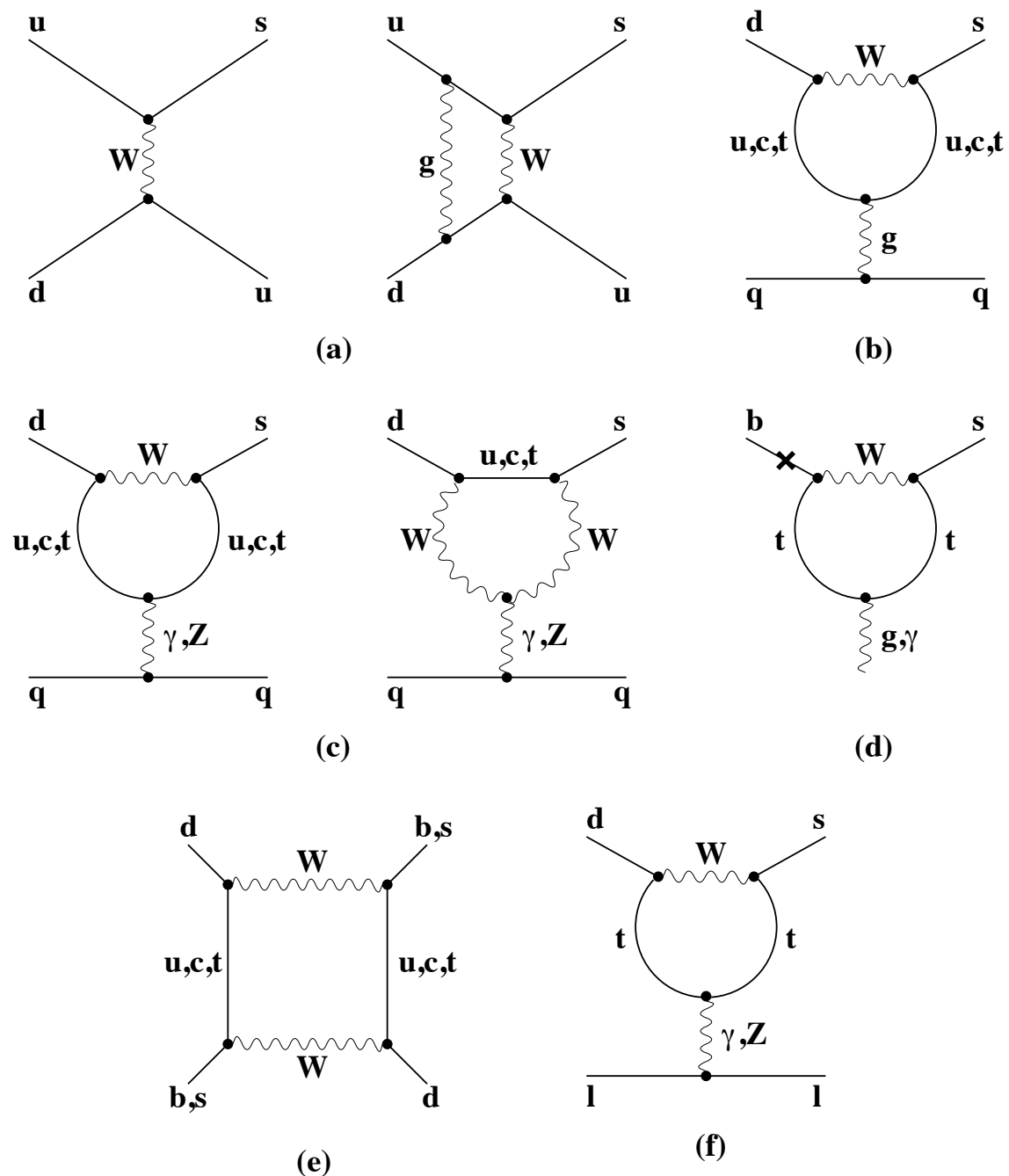
$$Q_{7\gamma} = \frac{-e}{8\pi^2} m_b \bar{s} \sigma_{\mu\nu} (1 + \gamma_5) F^{\mu\nu} b,$$

$$Q_{8g} = \frac{-g_s}{8\pi^2} m_b \bar{s} \sigma_{\mu\nu} (1 + \gamma_5) G^{\mu\nu} b,$$

# Weak interactions at low energies (flavor physics)

Example: Effective Lagrangian for  $b \rightarrow s$  FCNC transitions  
(see Buras lectures for a derivation)

SM diagrams involving virtual heavy-particle exchanges contributing to the low-energy effective weak Lagrangian



# Weak interactions at low energies (flavor physics)

---

Example: Effective Lagrangian for  $b \rightarrow s$  FCNC transitions  
(see Buras lectures for a derivation)

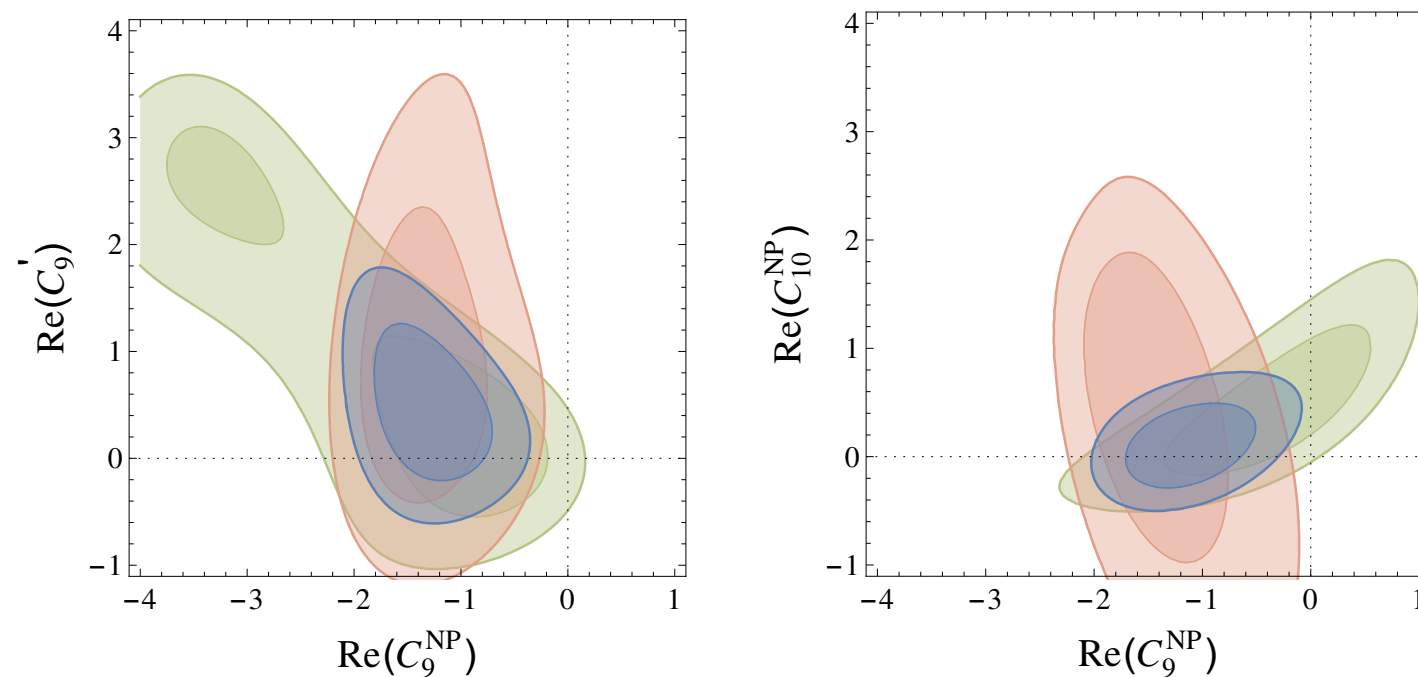
From the fact that the **leading operators** in the low-energy effective theory have **dimension 6**, it follows that the corresponding couplings are **irrelevant** and proportional to  $M_W^2$ , indeed:

$$\frac{G_F}{\sqrt{2}} = \frac{g_2^2}{8M_W^2}$$

The strong suppression of these contributions at low energies explains why we refer to these interactions as the **weak interactions**, even though the coupling constants of the  $SU(2)_L \otimes U(1)_Y$  electroweak interactions is about as large as the electromagnetic coupling constant

# Weak interactions at low energies (flavor physics)

A global analysis of experimental data on  $B \rightarrow X_s \gamma$ ,  $B \rightarrow K^* \gamma$ , and  $B \rightarrow K^{(*)} \mu^+ \mu^-$  decay distributions provides information about various operator coefficients (defined to vanish in SM):



Allowed regions in the  $\text{Re}(C_9^{\text{NP}})$ - $\text{Re}(C'_9)$  plane (left) and the  $\text{Re}(C_9^{\text{NP}})$ - $\text{Re}(C_{10}^{\text{NP}})$  plane (right). The blue contours correspond to the 1 and  $2\sigma$  best fit regions from the global fit. The green and red contours correspond to the 1 and  $2\sigma$  regions if only branching ratio data or only data on  $B \rightarrow K^* \mu^+ \mu^-$  angular observables is taken into account.

Altmannshofer, Straub:1411.3161

A first hint of New Physics?

$$\mathcal{H}_{\text{eff}} = -\frac{4 G_F}{\sqrt{2}} V_{tb} V_{ts}^* \frac{e^2}{16\pi^2} \sum_i (C_i O_i + C'_i O'_i) + \text{h.c.}$$

SM operators:

$$O_7 = \frac{m_b}{e} (\bar{s} \sigma_{\mu\nu} P_R b) F^{\mu\nu}$$

$$O_9 = (\bar{s} \gamma_\mu P_L b) (\bar{\ell} \gamma^\mu \ell)$$

$$O_{10} = (\bar{s} \gamma_\mu P_L b) (\bar{\ell} \gamma^\mu \gamma_5 \ell)$$

Opposite chirality operators:

$$O'_7 = \frac{m_b}{e} (\bar{s} \sigma_{\mu\nu} P_L b) F^{\mu\nu}$$

$$O'_9 = (\bar{s} \gamma_\mu P_R b) (\bar{\ell} \gamma^\mu \ell)$$

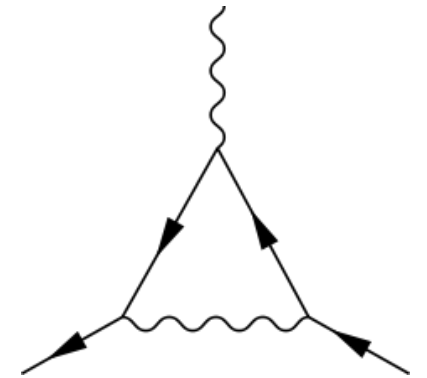
$$O'_{10} = (\bar{s} \gamma_\mu P_R b) (\bar{\ell} \gamma^\mu \gamma_5 \ell)$$

# Anomalous magnetic moment of the muon

---

In a celebrated calculation that was the birth of modern QFT, Schwinger computed the anomalous magnetic moment of the electron in 1948 and found:

$$\mu_e = \frac{g_e}{2m_e}, \text{ with } a_e = \frac{g_e - 2}{2} = \frac{\alpha}{2\pi} + \dots$$



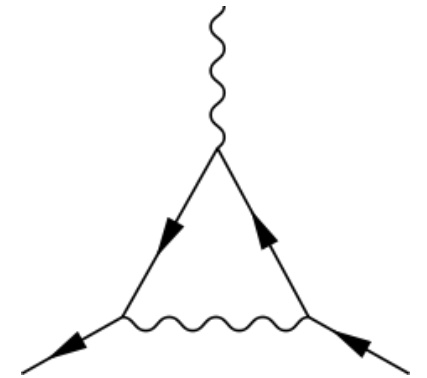
How will this result be affected if the SM is considered as an effective field theory?

# Anomalous magnetic moment of the muon

---

In a celebrated calculation that was the birth of modern QFT, Schwinger computed the anomalous magnetic moment of the electron in 1948 and found:

$$\mu_e = \frac{g_e}{2m_e}, \text{ with } a_e = \frac{g_e - 2}{2} = \frac{\alpha}{2\pi} + \dots$$



How will this result be affected if the SM is considered as an effective field theory?

Add **unique** dimension-5 operator ( $\delta = 5$ ,  $\gamma = -1$ ):

$$\frac{gv}{M^2} \bar{\psi} \sigma_{\mu\nu} F^{\mu\nu} \psi$$

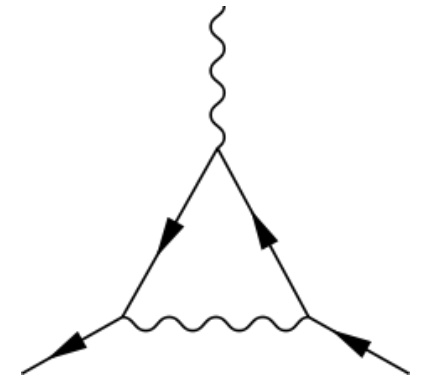
factor  $v$  required by EWSB

# Anomalous magnetic moment of the muon

---

In a celebrated calculation that was the birth of modern QFT, Schwinger computed the anomalous magnetic moment of the electron in 1948 and found:

$$\mu_e = \frac{g_e}{2m_e}, \text{ with } a_e = \frac{g_e - 2}{2} = \frac{\alpha}{2\pi} + \dots$$



This adds  $g/M$  to  $\mu_e$  and hence:

$$a_e = \frac{\alpha}{2\pi} + \frac{gm_e v}{M^2} + \dots$$

As long as  $M \gg m_e$  the additional term will be very small, and by comparing a measurement of  $\mu_e$  with theory we can constrain M

# Anomalous magnetic moment of the muon

---

Analogous discussion (with  $m_e$  replaced by  $m_\mu$ ) holds for the muon

In this case, there is presently a **3.6  $\sigma$  discrepancy** between theory and experiment:

$$a_\mu^{\text{SM}} - a_\mu^{\text{exp}} \approx -2.8 \cdot 10^{-9}$$

Interpreting this effect in terms of our irrelevant operator implies that:

$$M \sim \sqrt{g} \times 100 \text{ TeV}$$

↙ contains loop factor (small)

One of the best hints for BSM physics!



# Proton decay

---

Suppose you know the gauge symmetry  $SU(3)_c \otimes SU(2)_L \otimes U(1)_Y$  of the SM but nothing else (no GUTs). What could you say about proton decay?

The effective Lagrangian must contain at least **three quark fields** (change baryon number by 1 unit) and **one lepton field** (change lepton number by 1 unit)

Hence:

$$\mathcal{L}_{\text{proton decay}} \sim \frac{g}{M^2} qqq\ell$$

Since the lowest-dimension operators have dimension 6 (corresponding to  $\gamma_i = -2$ ), the proton can be made sufficiently long-lived by raising the fundamental scale  $M$  into the  $10^{16}$  GeV range

# Proton decay

---

Now imagine that you do not know about the existence of quarks (no one has seen any) but you do know about protons and pions

Then an effective Lagrangian giving proton decay could be:

$$\mathcal{L}_{\text{proton decay}} \sim g \pi \bar{\psi}_e \psi_p$$

This is a marginal operator, and hence proton decay would not be suppressed by any large mass scale!

In some sense, we see that the **longevity of the proton** provides a hint for a substructure of the proton: **replacing a fundamental field by a composite of several fields** raises the dimension of the operators and hence gives rise to additional suppression

# Proton decay

---

The same trick can be applied to other fine-tuning problems

For example, the hierarchy problem can be solved by supposing that the **Higgs boson is not an elementary scalar** particle but instead a **composite of a pair of elementary fermions**

If this is the case, then the Higgs mass term corresponds to a 4-fermion operator, which is irrelevant

This is the main idea of **composite Higgs** and **technicolor** theories

# Baryon and lepton number conservation

---

In the construction of the SM, the conservation of baryon and lepton number is **not imposed** as a condition

There are no corresponding  $U(1)$  symmetries of the Lagrangian

How can we understand that in nature we have not seen any hints of baryon- or lepton-number violating processes?

# Baryon and lepton number conservation

---

In the construction of the SM, the conservation of baryon and lepton number is **not imposed** as a condition

There are no corresponding  $U(1)$  symmetries of the Lagrangian

How can we understand that in nature we have not seen any hints of baryon- or lepton-number violating processes?

The answer is that **it is impossible to construct any relevant or marginal operator** that would respect the gauge symmetries of the SM and **violate baryon or lepton number!**

Hence, at the level of renormalizable interactions, baryon- and lepton-number conservation are **accidental symmetries** of the SM

# Higgs production at the LHC

---

The protons collided at the LHC contain only light quarks (u,d, and a little bit of s), which in the SM have negligible couplings to the Higgs boson, and gluons, which do not couple to the Higgs boson at all

How, then, is the Higgs boson produced in pp collisions at the LHC?

# Higgs production at the LHC

---

The protons collided at the LHC contain only light quarks (u,d, and a little bit of s), which in the SM have negligible couplings to the Higgs boson, and gluons, which do not couple to the Higgs boson at all

How, then, is the Higgs boson produced in pp collisions at the LHC?

We can gain insight by assuming (as seems to be the case) that the Higgs boson is lighter than the top quark

We can then construct an effective low-energy theory for Higgs physics, in which the top quark is integrated out



# Higgs production at the LHC

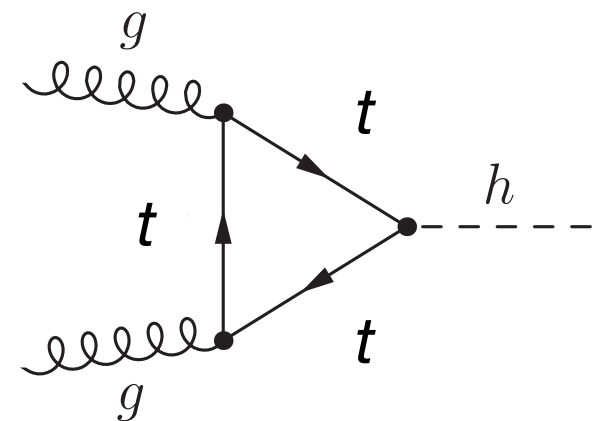
---

In this effective low-energy theory, direct couplings of the Higgs boson to pairs of gluons and photons arise at the level of **irrelevant dimension-5 operators**, with coefficients that scale like  $1/m_t$ , e.g.:

$$\mathcal{L}_{hgg} = \frac{y_t}{\sqrt{2}m_t} \frac{\alpha_s}{12\pi} h G_{\mu\nu}^a G^{\mu\nu,a}$$

These operators appear first at one-loop order, via the exchange of a virtual top-quark

The **effective hgg interaction** provides the dominant production mechanism for the Higgs boson in **gluon-gluon fusion** at the LHC



# Summary

---

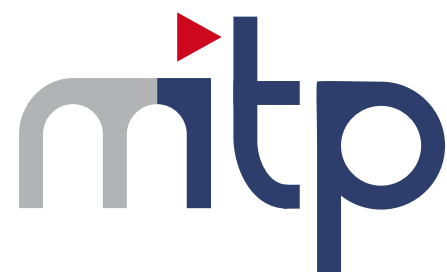
Effective field theories are a very powerful tool in quantum field theory

They are of great **practical use**, but also provide the **conceptual tools** to understand scale separation (factorization) and renormalization in a physical and systematic way

Effective field theories are abundant, since any QFT can be considered as an effective low-energy theory of some more fundamental theory, which is often not yet known

Because of this fact, effective field theories provide the tools to perform **indirect searches for new physics** beyond the Standard Model





## Mainz Institute for Theoretical Physics

### SCIENTIFIC PROGRAMS JGU Campus Mainz

#### Effective Theories and Dark Matter

Vincenzo Cirigliano LANL, Richard J. Hill Univ. Chicago, Achim Schwenk, TU Darmstadt, Tim M.P. Tait UC Irvine

March 16-27, 2015

#### Amplitudes, Motives and Beyond

Francis Brown IHES Paris, Marcus Spradlin Brown Univ., Don Zagier MPI-M Bonn, Stefan Weinzierl JGU Mainz, Stefan Müller-Stach JGU Mainz

May 26-June 12, 2015

#### Higher Orders and Jets at the LHC

Matteo Cacciari LPHE Paris, Paolo Nason INFN Milan, Giulia Zanderighi CERN & Univ. Oxford

June 29-July 17, 2015

#### Crossroads of Neutrino Physics

Steen Hannestad Aarhus Univ., Patrick Huber Virginia Tech, Alexei Smirnov MPI-K Heidelberg, Joachim Kopp JGU Mainz

July 20-August 14, 2015

#### Fundamental Parameters from Lattice QCD

Gilberto Colangelo Bern Univ., Heiko Lacker HU Berlin, Georg von Hippel JGU Mainz, Hartmut Wittig JGU Mainz

August 31-September 11, 2015

#### Stringy Geometry

Eric Bergshoeff Univ. Groningen, Gianfranco Pradisi University of Rome „Tor Vergata“, Fabio Riccioni INFN Rome „La Sapienza“, Gabriele Honecker JGU Mainz

September 14-25, 2015

### TOPICAL WORKSHOPS JGU Campus Mainz

#### Challenges in Semileptonic B Decays

Paolo Gambino Univ. Turin, Marcello Rotondo INFN Padua, Christoph Schwanda ÖAW Vienna, Andreas Kronfeld Fermilab, Sascha Turczyk JGU Mainz

April 20-24, 2015

#### Higgs Pair Production at Colliders

Daniel de Florian Univ. Buenos Aires, Christophe Grojean CERN, Fabio Maltoni Univ. Leuven, Alessandro Nisati INFN Rome, José Zurita JGU Mainz

April 27-30 2015

#### The Ultra-Light Frontier

Surjeet Rajendran Univ. Stanford, Dmitry Budker JGU Mainz

June 15-19, 2015

#### Quantum Vacuum and Gravitation

Manuel Asorey Univ. Zaragoza, Emil Mottola LANL, Ilya Shapiro Univ. Juiz de Fora, Andreas Wipf Univ. Jena

June 22-26, 2015

For more details: <http://www.mitp.uni-mainz.de>

Applications via <http://indico.mitp.uni-mainz.de>



[www.mitp.uni-mainz.de](http://www.mitp.uni-mainz.de)



Mainz Institute for Theoretical Physics  
PRISMA Cluster of Excellence  
Johannes Gutenberg-Universität Mainz, Germany